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Title: Design of Intelligent Controller for Inverted Pendulum

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ABSTRACT

Stability is one of the most important aspects in the traditional knowledge of Automatic Control. The most widely used control strategy in industries is the proportional integral derivative (PID) controller. The PID controllers are popular as they can perform robustly in a wide range of operating conditions and due to their functional simplicity. Our research is to design "An Intelligent (Fuzzy Logic) controller" using simple approach & smaller rule set and to implement the controller on inverted pendulum system to control the velocity and angle of the inverted pendulum system. Performance of Controller will be demonstrated through Simulation. Performance analysis will show the effectiveness of the proposed Fuzzy Logic Controller as compared to the PID controller.

Keywords: Stability, Fuzzy control, Pendulum, Proportion, Integral and Derivative (PID)

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Introduction

Fuzzy Logic Controller is one of the most important applications of fuzzy rule based system that models the human decision processing with a collection of fuzzy rules and sets which is essential for a fuzzy logic controller to perform at the desired level.

Fuzzy Logic Controllers have been successfully and widely applied to various fields for decades. Normally, a typical fuzzy logic system contains fuzzifier, rules, inference engine and defuzzifier.

Fuzzy Logic incorporates a simple, rule-based IF X AND Y THEN Z approach to a solving control problem rather than attempting to model a system mathematically. The FL model is empirically-based, relying on an operator's experience rather than their technical understanding of the system. For example, rather than dealing with temperature control in terms such as "SP =500F", "T <1000F", or "210C <TEMP <220C", terms like "IF (process is too cool) AND (process is getting colder) THEN (add heat to the process)" or "IF (process is too hot) AND (process is heating rapidly) THEN (cool the process quickly)" are used. These terms are imprecise and yet very descriptive of what must actually happen.

A fuzzy control system is a control system based on fuzzy logic in a mathematical system that analyzes analog input values in terms of logical variables that take on continuous values between 0 and 1, in contrast to classical or digital logic, which operates on discrete values of either 1 or 0 (true or false, respectively). Fuzzy controllers are very simple conceptually. They consist of an input stage, a processing stage, and an output stage. The input stage maps sensor or other inputs, such as switches, thumbwheels, and so on, to the appropriate membership functions and truth values. The processing stage invokes each appropriate rule and generates a result for each, then combines the results of the rules. Finally, the output stage converts the combined result back into a specific control output value.

A fuzzy controller uses fuzzy sets to convert values of its input variables to fuzzy set memberships. These fuzzy sets are called input fuzzy sets. Obviously, the analytical input-output structure of a fuzzy controller is determined by input fuzzy sets along with other components such as fuzzy rules and fuzzy AND operator in the rules. Many different types of input fuzzy sets exist in the literature. Nevertheless, trapezoidal type, triangular type, Gaussian type, and bell type are among the most often used ones. As for fuzzy AND operator, only two types are popular: product AND operator and Zadeh AND operator. Deriving the analytical structure of a fuzzy controller that uses the former operator is not very difficult, regardless of shape of input fuzzy sets. On the other hand, obtaining the analytical structure involving Zadeh fuzzy AND operator is challenging even for triangular input fuzzy sets, which are the simplest fuzzy sets, relatively speaking. This is the case partially due to the necessity of analytical execution of Zadeh AND operator.

A. Type-1 Fuzzy Logic Control

Type-1 FLCs are both intuitive and numerical systems that map crisp inputs to a crisp output. Every FLC is associated with a set of rules with meaningful linguistic interpretations, such as R^l : If x_1 is $F1^l$ and x_2 is $F2^l$ and … and $x_n$is Fn^l Then w is G^l which can be obtained either from numerical data, or experts familiar with the problem at hand. Based on this kind of statement, actions are combined with rules in an antecedent/consequent format, and then aggregated according to approximate reasoning theory, to produce a nonlinear mapping from input space $U = U_1 xU_2 x...U_n$ …. to the output space W , where F_k k $k=1,2,\ldots\ldots n$ are the antecedent type-1 membership functions, and G^l W is the consequent type-1 membership function. The input linguistic variables are denoted by u_k , k =1,2,...,n and the output linguistic variable is denoted by w A Fuzzy Logic System (FLS) as the kernel of a FLC consist of four basic elements (Fig. 1): the *type-1*fuzzifier, the fuzzy rule-base, the inference engine, and the type-1 defuzzifier. The fuzzy rule-base is a collection of rules in the form of R^l , which are combined in the inference engine, to produce a fuzzy output. The type-1 fuzzifier maps the crisp input into type-1 fuzzy sets, which are subsequently used as inputs to the inference engine, whereas the type-1 defuzzifier maps the type-1fuzzy sets produced by the inference engine into crisp numbers.

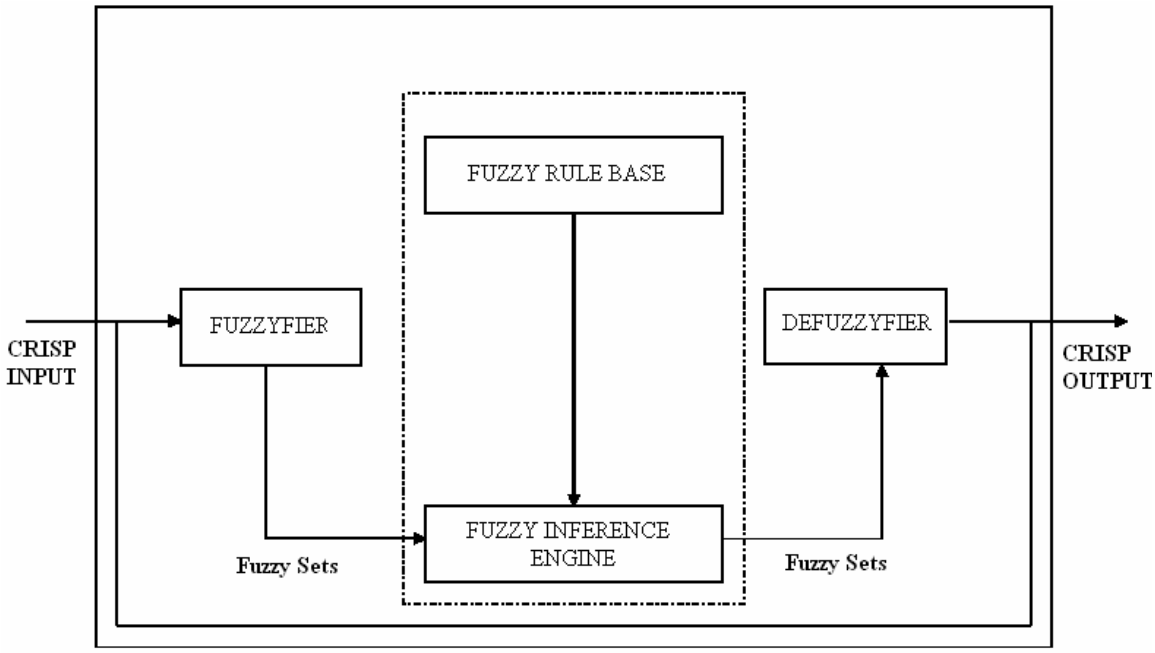

Fig.1. Structure of type-1 fuzzy logic system

Fuzzy sets can be interpreted as membership functions u_x that associate with each element x of the universe of discourse, U , a number $u_X(x)$ in the interval [0,1]:

$$u_x : U \rightarrow [0,1] \qquad (1)$$

B. Type-2 Fuzzy Logic Control

As the type-1 fuzzy set, the concept of type-2 fuzzy set was introduced by Zadeh as an extension of the concept of an ordinary fuzzy set. A FLS described using at least one type-2 fuzzy set is called a type-2 FLS. Type-1 FLSs are unable to directly handle rule uncertainties, because they use type-1 fuzzy sets that are certain. On the other hand, type-2 FLSs are very useful in circumstances where it is difficult to determine an exact, and measurement uncertainties. It is known that type-2 fuzzy set let us to model and to minimize the effects of uncertainties in rule-based FLS. Similar to a type-1 FLS, a type-2 FLS includes type-2 fuzzifier, rule-base, inference engine and substitutes the defuzzifier by the output processor. The output processor includes a type-reducer and a type-2 defuzzifier; it generates a type-1 fuzzy set output (from the type reducer) or a crisp number (from the defuzzifier). A type- 2 FLS is again characterized by IF-THEN rules, but their antecedents of consequent sets are now type-2. Type-2 FLSs, can be used when the circumstances are too uncertain to determine exact membership grades. A model of a type-2 FLS is shown in Fig.2

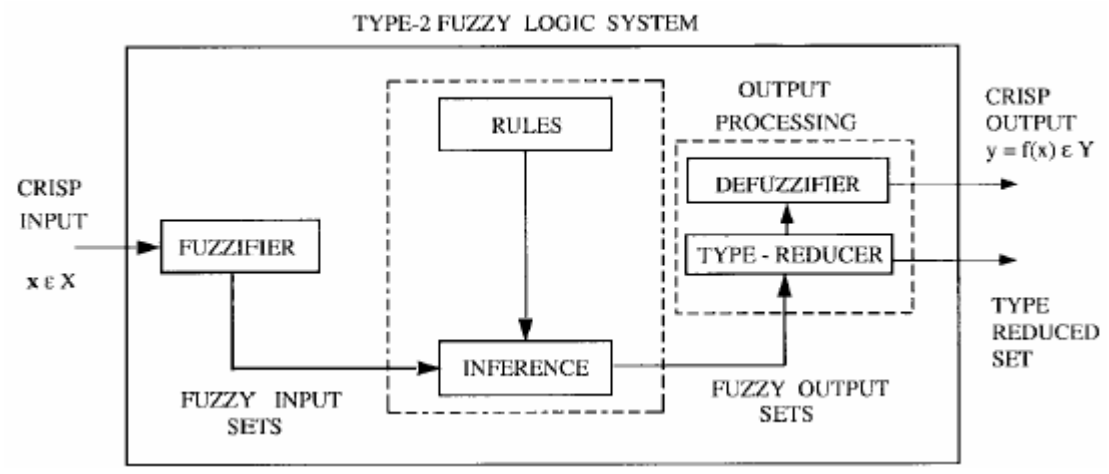

Fig.2. Structure of type-2 fuzzy logic system

In the case of the implementation of the type-2 FLCs, we have the same characteristics as in type-1 FLC, but we use type-2 fuzzy sets as membership functions for the inputs and for the outputs. Fig. 3 shows the structure of a control loop with a FLC.

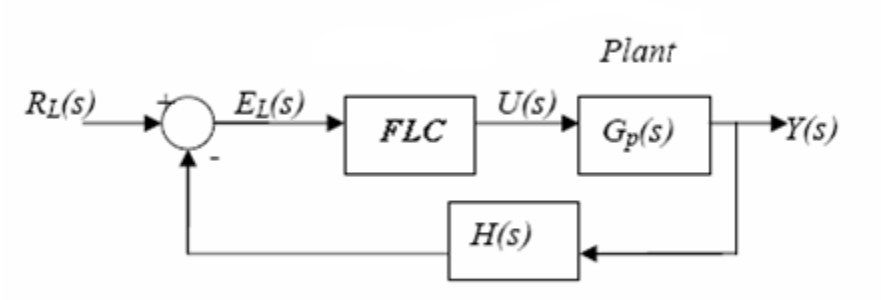

Fig3. Fuzzy control loop

Inverted Pendulum System

The inverted pendulum problem is one of the most important problems in dynamics and has been studied excessively in the control literature. The system is nonlinear, unstable, non minimum phase and under actuated. It is also known that an inverted pendulum and cart (IPC) system is a complex nonlinear system. The cart with an inverted pendulum, shown below, is "bumped" with an impulse force, F. First the dynamic equations of motion for the system are found, then linearize about the pendulum's angle, theta = 0 (in other words, assume that pendulum does not move more than a few degrees away from the vertical, chosen to be at an angle of 0). A controller is used to satisfy all of the required design requirements.

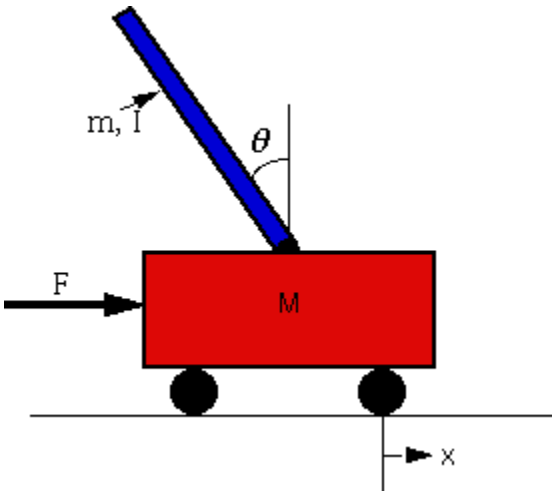

Fig 4. Inverted Pendulum and Cart System

M	mass of the cart
m	mass of the pendulum
b	friction of the cart
l	length to pendulum center of mass
I	inertia of the pendulum
F	force applied to the cart
x	cart position coordinate
theta	pendulum angle from vertical

State of art

The Margaliot approach for the design of FLC is proved to be valid [25] for both type1 and Type 2 FLC. On type 2 FLC membership functions definition domain of the FOU can be changed without loss of stability of the controller.

In [7] it has been proven that the general T2 fuzzy systems can uniformly approximate any real continuous function on a compact domain or arbitrary accuracy.
A general technique is developed in [13] for deriving analytical Input-Output structure for fuzzy controllers either Mamdani type or TS type, using Zadeh fuzzy AND operator and arbitrary types of input fuzzy sets.

A function based evaluation approach is proposed in [4] for a systematic study of fuzzy PID like controllers.

In [22] a new approach to tune the control gains of a PD controller has been outlined. The proposed novel FLC uses the rule premises and consequences.

The PID controller is used for a wide range of problems like motor drives, automotive, flight control, instrumentation etc. PID controllers provide robust and reliable performance for most systems if the PID parameters are tuned properly. Various tuning methods are explained in [17], [26] and [18].

Since the field of Fuzzy control has been making rapid progress in recent years. Fuzzy logic control has been widely exploited for nonlinear, high order & time delay system [26]. This paper has two main contributions. Firstly, a PID controller has been designed for higher order system using Ziegler-Nichols frequency response method and its performance has been observed. The Ziegler Nichols tuned controller parameters are fine tuned to get satisfactory closed loop performance. Secondly, for the same system a fuzzy logic controller has been proposed with simple approach and smaller number of rules (four rules) as it gives the same performance. Simulation results for a higher order system have been demonstrated. A performance comparison between Ziegler Nichols tuned PID controller, fine-tuned PID controller and the proposed fuzzy logic controller is presented.

In his paper Kazemia, H.B [18] was concerned with two types of fuzzy PID controllers, the rule-based fuzzy PID controller and the learning fuzzy PID controller. For the rule based fuzzy PID controller, the PID gains were firstly tuned using a conventional tuning method The rule buffer of the rule-based fuzzy then readjusts the PID gains during the system operation. The learning fuzzy PID controller not only readjusts the PID gains, it also creates and updates the rules in the de-buffer automatically during the application of the learning fuzzy PID controller to the process. The rule based fuzzy PID controller and the learning fuzzy PID controller were both applied to a non-linear revolute joint robot-arm for path tracking experiments to trace a square waveform. The results of the experiments for the rule-based fuzzy PID controller and the learning fuzzy controller were compared with a conventional P ID controller in order to evaluate the results. It was concluded that in general, the learning fuzzy PID controller performs better in comparison with the rule based fuzzy PID controller. This is because the rules in the rule buffer are updated and changed constantly during the application of the learning fuzzy PID controller to the process. The rule-based fuzzy PID controller and the learning fuzzy PID controller both produce a better process output response than the PID controller, in the presence of noise and time variant dynamics. For the rule-based fuzzy PID controller and the learning fuzzy PID controller, further increase in the amplitude and frequency of the square waveform have little effect on the process output as opposed to the conventional PID controller.

The PID fuzzy controller by Qingchun Li; Deyao Shen [27] is fallen into one dimension, two-dimension and three-dimension one in terms of structure, among which the two-dimension PID fuzzy controller is the most popular one. In this paper, proposed an incremental fuzzy PD+ fuzzy ID controller which input variables are error $e(kt)$, change of error $ec(kt)$ and rate of change of error $ed(kt)$, analyzed its structure, and obtain its formulas in all regions. The fuzzy PID controller has the characteristic of a nonlinear controller with time varying PID gains. While its design is easy, its structure is simple and its control precision is high. The most important advantage of the fuzzy PID controller is that it is possible to design a control system whose plant dynamics is not known, by only using the input/output information. Also, a linear PID controller can naturally be derived under the design procedure of the fuzzy PID controller, although plant dynamics is not known. The simulation research result shows that its control feature is better than that of the fuzzy PI and the conventional PID controller.

A novel T2 adaptive fuzzy controller [6] is developed by Haibo Zhou; Hao Ying; Ji'an Duan. A series of simulation using the nonlinear inverted pendulum model are conducted. The simulation results show that the controller achieves good control performance and guarantees the system stability. When coping with disturbance and noise the controller can maintain good robust control performance.

Control performance comparison between a type- 1 fuzzy controller (T1-FC) and a comparable type-2 fuzzy controller (T2-FC) was carried out by Xinyu Du; Hao Ying using computer simulation [34]. The objective was to study whether T2 fuzzy control always had a control performance advantage over its T1 counterpart as claimed in some simulation-based reports. A genetic algorithm was used to optimize the T1-FC and the

T2-FCs that control process models of three different types (i.e., linear, linear with a time-delay, and nonlinear). Controllers' robustness against model parameter variation and capabilities of dealing with random noise were compared as well. The simulation results show that different settings result in different comparison outcomes: (1) the T1-FC and the T2-FC performed (almost) identically, and (2) the T2-FC outperformed its T1 counterpart, and (3) the T1- FC was superior. These results are theoretically sensible because from the controllers' input-output mapping standpoint, their ability to produce continuous nonlinear control functions should be similar and no inherent advantage likely exists. Thus, one controller can appear to be better than, worse than, or equal to its counterpart depending on the specific configuration of the whole control system. Consequently, no one should claim that T2 fuzzy control is generally better than T1 fuzzy control.

Biglarbegian, M.; Melek, W.W.; Mendel, J.M.[3] proposed a novel inference mechanism for an interval type-2 Takagi–Sugeno–Kang fuzzy logic control system (IT2 TSK FLCS) when antecedents are type-2 fuzzy sets and consequents are crisp numbers (A2-C0). The proposed inference mechanism has a closed form which makes it more feasible to analyze the stability of this FLCS. This paper focuses on control applications for the following cases: 1) Both plant and controller use A2-C0 TSK models, and 2) the plant uses type-1 Takagi–Sugeno (TS) and the controller uses IT2 TS models. In both cases, sufficient stability conditions for the stability of the closed-loop system are derived. Furthermore, novel linear-matrix inequality- based algorithms are developed for satisfying the stability conditions. Numerical analyses are included which validate the effectiveness of the new inference methods. Case studies reveal that an IT2 TS FLCS using the proposed inference engine clearly outperforms its type-1 TSK counterpart. Moreover, due to the simple nature of the proposed inference engine, it is easy to implement in real-time control systems. The methods presented in this paper lay the mathematical foundations for analyzing the stability and facilitating the design of stabilizing controllers of IT2 TSK FLCSs and IT2 TS FLCSs with significantly improved performance over type-1 approaches.

Lam, H.K.; Seneviratne, L.D [21] proposed a fuzzy controller to realize the tracking control based on the fuzzy-model-based control approach. Stability conditions have been derived to guarantee the system stability based on Lyapunov approach. The finding of the feedback gains and the guarantee of the tracking performance have been formulated as a GEVP which can be solved numerically and efficiently using convex programming techniques. An application example dealing with the tracking control of a two-link robot arm has been given to demonstrate the effectiveness of the proposed approach

In this paper Castillo, O.; Cazarez, N.; Melin, P e proved [4] Margaliot approach for the design of FLC is valid for both, Type-1 and Type-2 Fuzzy Logic Controllers. On Type-2 FLC's membership functions we can perturb or change the definition domain of the FOU without loss of stability of the controller while we can give a proportional gain to the control loop gains. To example of the inverted pendulum, the stability holds extending the FOU on the domain [0, 1], Stability is one of the more important aspects in the traditional knowledge of Automatic Control.

Fuzzy systems using type-2 fuzzy sets and fuzzy logic are important. In his paper, Hao Ying [9] constructively proved that a general class of Mamdani interval type-2 fuzzy systems is universal approximator. This result lays a theoretical foundation for the fuzzy systems development. The components of the general Mamdani interval T2 fuzzy systems studied in this paper are quite arbitrary in terms of input fuzzy sets, fuzzy AND and OR operators as well as fuzzy inference. Utilizing the same proof approach that was developed for the general Mamdani T1 fuzzy systems, it is constructively proven that the general T2 fuzzy systems can uniformly approximate any real continuous function on a compact domain to arbitrary accuracy.

In the paper by Takacs, M [33] a new method for on-line determination of the gains of a PD controller by using a separate new type fuzzy logic controller is given. Based on the linearity of the control law the possibility measures of the rules of the FLC were introduced. The calculation of these possibility measures offers new horizons for the rule base construction. The proposed new FLC model restricts the Mamdani type rule consequences to possibility domain. In order to verify the performance of the proposed controller simulation has been carried out. It was concluded that in case of the application of generalized t-norms in the rules and in the inference mechanism the new method provide better performance than the conventional Mamdani type controller.

A fuzzy controller uses either Zadeh or product fuzzy AND operator, with the former being more frequently used than the latter. A general technique for rigorously deriving analytical input–output structure for fuzzy controllers is developed by Hao Ying, [10] that uses Zadeh fuzzy AND operator and arbitrary types of input fuzzy sets. Some general necessary and sufficient conditions relating the shape of input fuzzy sets to the shape of input space divisions are also developed. The relationships as well as the technique are applicable to any fuzzy controllers that uses Zadeh AND operator, regardless of type of fuzzy rules, type of output fuzzy sets (for Mamdani rules) or type of functions in rule consequents (TS rules), type of fuzzy inference, and type of defuzzifier. They are also directly applicable to comparable fuzzy models and fuzzy expert systems. Furthermore, the design issue of selecting input fuzzy set type is examined by examining the new input–output structures that was derived. It was found that the fuzzy controllers using the trapezoidal/triangular input fuzzy sets can be understood (and possibly analyzed and designed) most sensibly and easily in the context of control theory. Consequently, it is recommended that the trapezoidal/triangular fuzzy sets be used as the first choice if the understanding of the input–output relation of a fuzzy controller is important.

A function-based evaluation approach is proposed for a systematic study of fuzzy proportional-integral-derivative (PID)-like controllers by Bao-Gang Hu; Mann, G.K.I.; Gosine, R.G[4]. This approach is applied for deriving process-independent design guidelines from addressing two issues: simplicity and nonlinearity. To examine the simplicity of fuzzy PID controllers, it was concluded that direct-action controllers exhibit simpler design properties than gain-scheduling controllers. Then, the inference structure was evaluated of direct-action controllers in five criteria: control-action composition, input coupling, gain dependency, gain-rule change, and rule/parameter growth. Three types of fuzzy PID controllers, using one-, two- and three-input inference structures, are

analyzed. The results, according to the criteria, demonstrate some shortcomings in the Mamdani's two-input controllers. For keeping the simplicity feature like a linear PID controller, a one-input fuzzy PID controller with "one-to-three" mapping inference engine is recommended.

Three evaluation approaches in nonlinear approximation study are discussed: function-estimation-based, generalization-capability-based and nonlinearity-variation-based approximations. The present study focuses on the last approach. A nonlinearity evaluation is then performed for several one-input fuzzy PID controllers based on two measures: nonlinearity variation index (NVI) and linearity approximation index (LAI). Using these quantitative indices, one can make a reasonable selection of fuzzy reasoning mechanisms and membership functions without requiring any process information. From the study it was observed that the Zadeh-Mamdani's "max-min-gravity" (MMG) scheme produces the highest score in terms of nonlinearity variations, which is superior to other schemes, such as Mizumoto's "product-sum-gravity" (PSG) and "Takagi–Sugeno–Kang" (TSK) schemes.

Hao Ying [11] developed a general technique for deriving analytical input-output structure for fuzzy controllers, either Mamdani type or TS type, that use Zadeh fuzzy AND operator and arbitrary types of input fuzzy sets. The general structure-deriving technique are applicable to any fuzzy controllers that use Zadeh AND operator, regardless of type of fuzzy rules, type of output fuzzy sets in the case of Mamdani fuzzy rules and type of functions as rule consequent in the case of TS rules, type of fuzzy inference, and type of defuzzifier.

S.R.Vaishnav, Z.J. Khan [31] presented an overview of PID controller, design of PID controller using Z-N technique and design of fuzzy logic controller for higher order system. Simulation results using MATLAB / SIMULINK are discussed for Ziegler Nichols tuned PID controller, fine tuned PID controller and the Fuzzy logic controller. Ziegler Nichols technique gives high overshoot and settling time with zero steady state error. Initial controller parameters obtained using Z-N formula need to be adjusted repeatedly through computer simulation to get satisfactory performance. Fine tuned PID controller gives zero steady state error and smaller overshoot and settling time than Ziegler Nichols tuned PID controller. The Fuzzy Logic controller gives no overshoot, zero steady state error and smaller settling time than obtained using Ziegler Nichols tuned PID controller and fine tuned PID controller. The simulation results confirms that the proposed. Fuzzy logic controller with simple design approach and smaller rule base can provide better performance comparing with the Ziegler Nichols tuned PID controller and fine tuned PID controller.

Ming-Ying Hsiao; Li, T.-H.S [24] proposed a novel interval type-2 fuzzy logic controller (IT2FLC) for interval system .A design scheme of IT2FLC is set up. The proposed FLC can be applied to a wide range variation of parameters of interval linear plant effectively. For better understanding the performance of IT2FLC, numerous shapes of membership functions and variation of rule numbers are evaluated for simulation thoroughly. All the simulation results demonstrate the feasibility and effectiveness of the developed IT2FLC.

It shows that the obvious distinction between outputs of IT2FLC and TIFLC is occurred as fewer rules are chosen, and IT2FLC performs better response than that of TlFLC.

Hagras, H [5] presented the type-2 FLC and highlighted its benefits, especially in highly uncertain environments. It has been shown that as the level of imprecision and uncertainty increases, the type-2 FLC will provide a powerful paradigm to handle the high level of uncertainties present in real-world environments. It has been also shown in various applications that the type-2 FLCs have given very good and smooth responses that have always outperformed their type-1 counterparts. Thus, using a type-2 FLC in real-world applications can be a better choice than type-1 FLCs since the amount of uncertainty in real systems most of the time is difficult to estimate.

In his paper Khan, A.A.; Rapal, N [20] proposed a fuzzy proportional integral derivative (PID) controller which can be tuned by carrying the tuning rules from PID domain to fuzzy domain. As a nonlinear controller can control a nonlinear process more efficiently, fuzzy controller can provide better performance in terms of rise time and smaller overshoot. The proposed controller is evaluated using some simulations. The simulation results show that both the transitory performance and the steady state performance are better than that of conventional PID controller.

To enhance the PID-type fuzzy controller performance for the control of rigid planar robot manipulators, in their paper Amer, A. F.; Sallam, E. A.; Elawady, W. M.;[1] proposed a fuzzy pre-compensation of a fuzzy self tuning fuzzy PID controller . The proposed control scheme consists of a fuzzy logic-based pre-compensator followed by a fuzzy self tuning fuzzy PID controller. In the fuzzy self tuning fuzzy PID controller, a supervisory hierarchical fuzzy controller (SHFC) is used for tuning the input scaling factors of the fuzzy PID controller according to the actual tracking position error and the actual tracking velocity error. Numerical simulations using the dynamic model of a three DOF planar rigid robot manipulator with uncertainties show the effectiveness of the approach in set point tracking problems. Results showed that the proposed controller has superior performance compared to a convention fuzzy PID controller.

Mann, G.K.I.; Hu, B.-G.; Gosine, R.G.; [23] presented in their paper that a fuzzy controller output surface based on linearity defined fuzzy regions in a linguistic error state space is used to define a general two input linear-like fuzzy logic controller (LLFLC). The output behavior of a LLFLC is generated using triangular membership functions to create a linear-like controller output surface in the normalized error state space and its properties are thus identified. A non-fuzzy equivalent linear controller (ELC) is established to generate a truly linear controller output surface. Two input linear-like fuzzy PI and PID control implementations and their ELC actions are presented. A first-order temperature control process with transport delay is used to test the performance of the above fuzzy and non-fuzzy, controllers. Fuzzy PI and PID controllers and their ELC structures are tested with different fuzzy resolutions. Results show that a LLFLC with fine fuzzy resolution and a non-fuzzy ELC behave similarly. As well, a

LLFLC with coarse fuzzy resolution has the benefit of the anti-symmetric, non-linearity property of the fuzzy controller output surface to produce better performance. Overall, a linear-like fuzzy PID controller with coarse fuzzy resolution gives the best performance.

Objectives

1) To design an Intelligent Controller using Fuzzy logic Type 1 and Type 2.(for a system)
2) To compare the performance of both the controllers with existing design on the system chosen.

Methodology

A PID controller is designed using Ziegler-Nichols (ZN) technique. An Intelligent logic controller using simple approach & smaller rule set is proposed. Simulation results using MAT LAB/SIMULINK will be demonstrated. Effectiveness of the proposed Intelligent controller as compared to the ZN tuned PID controller will be shown using performance analysis.

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